$\bar{L}$ = overall liquid stream molar flow rate defined by Eq. 8
M = number of components of the mixture
N = number of stages, including reboiler and condenser
P = pressure
Q = rate of heat leaving the stage
s = number of specification equations
T = temperature
U = liquid side stream molar flow rate
V = vapor stream molar flow rate
W = vapor side stream molar flow rate
x = liquid mole fraction
$\bar{x}$ = overall liquid mole fraction defined by Eq. 10
y = vapor mole fraction
z = feed mole fraction

Greek Letters

η = phase splitting parameter defined by Eq. 9
λ = parameter in Eq. 20

Superscript

$'$ = second liquid phase

Subscripts

i = component index
j = stage index

LITERATURE CITED

Ball, W. and F. Beitrag, "Computer Program for Distillation," 44th National Meeting of the AIChE, New Orleans (1961).

Block, U. and B. Hegner, "Development and Application of a Simulation Model for Three-Phase Distillation," *AIChE J.*, **22**, 582 (1976).

Boston, J. F. and S. L. Sullivan, "An Improved Algorithm for Solving Mass Balance Equations in Multistage Separation Processes," *Can. J. Chem. Eng.*, **50**, 663 (1972).

Buzzi Ferraris, G., "Interlinked, Multistaged Separators with Non-Standard specifications Solved by the Newton-Raphson Method," *AIChE J.*, **27**, 163 (1981).

Buzzi Ferraris, G. and A. Casapollo, "Risoluzione di Sistemi di Equazioni Algebriche non Lineari e loro Applicazione a Problemi dell'Ingegneria Chimica," *Ing. Chim. Ital.*, **8**, 261 (1972).

Friday, J. R. and B. D. Smith, "An Analysis of the Equilibrium Stage Separation Problem-Formulation and Convergence," *AIChE J.*, **10**, 698 (1964).

Henley, E. J. and E. M. Rosen, *Material and Energy Balance Computations*, J. Wiley & Sons Inc., New York (1969).

Holland, C. D., *Fundamentals and Modeling of Separation Process-Absorption, Distillation, Evaporation, and Extraction*, Prentice-Hall, Englewood Cliffs, NJ (1975).

Naphtali, L. and D. S. Sandholm, "Multicomponent Separation Calculation by Linearization," *AIChE J.*, **17**, 148 (1971).

Wang, J. C. and G. E. Henke, "Tridiagonal Matrix for Distillation," *Hydrocarbon Process*, **45**, 155 (1966).

Manuscript received June 3, 1980; revision received October 30, and accepted November 21, 1980.

Collection Efficiency of Cyclone Separators

P. W. DIETZ

General Electric Company
Corporate Research and Development
Schenectady, NY 12301

Based on terLinden's (A. J. terLinden, "Investigations into Cyclone Dust Collectors," IME Proc., 160, 233, 1949) experimental observations, a three-region model is proposed for the fluid flow in a cyclone. Within each region, turbulence is assumed to promote mixing of the suspended particles. Incorporation of this mixing concept into the three-region model allows an analytic expression for the collection efficiency of the cyclone to be developed.

The theoretical result is compared with data obtained in the high temperature, high pressure exhaust from a pressurized fluidized bed combustor.

SCOPE

In the present paper, an analytic expression is developed for the efficiency of reverse flow cyclone separators. To develop this equation, a simple model for the gas flow within the cyclone is proposed. Based on this model, the cyclone is separated into three regions: an inlet region, a downflow region, and an upflow region. Within each of the regions, radial particle-transport is assumed to be dominated by turbulent mixing. Consequently, a simple equation for conservation of particles can be stated and the expression for cyclone efficiency results from the solution of the equation subject to appropriate boundary conditions.

Comparisons of the resultant expression with data from the second stage cyclone at Exxon's PFB miniplant yield satisfactory agreement.

CONCLUSIONS AND SIGNIFICANCE

With the growing concern for the environmental effects of particulate pollution, it becomes increasingly important to be able to design optimized pollution control devices. For instance, although measurements of the fractional efficiency of cyclone separators can be used to predict the performance of an existing design in new applications, the efficiency of new designs cannot be predicted based on current theories. In the present paper, a new, analytic model is developed for the performance of cyclone separators which may provide the needed design tool.

0001-1541-81-4962-0808-\$2.00.

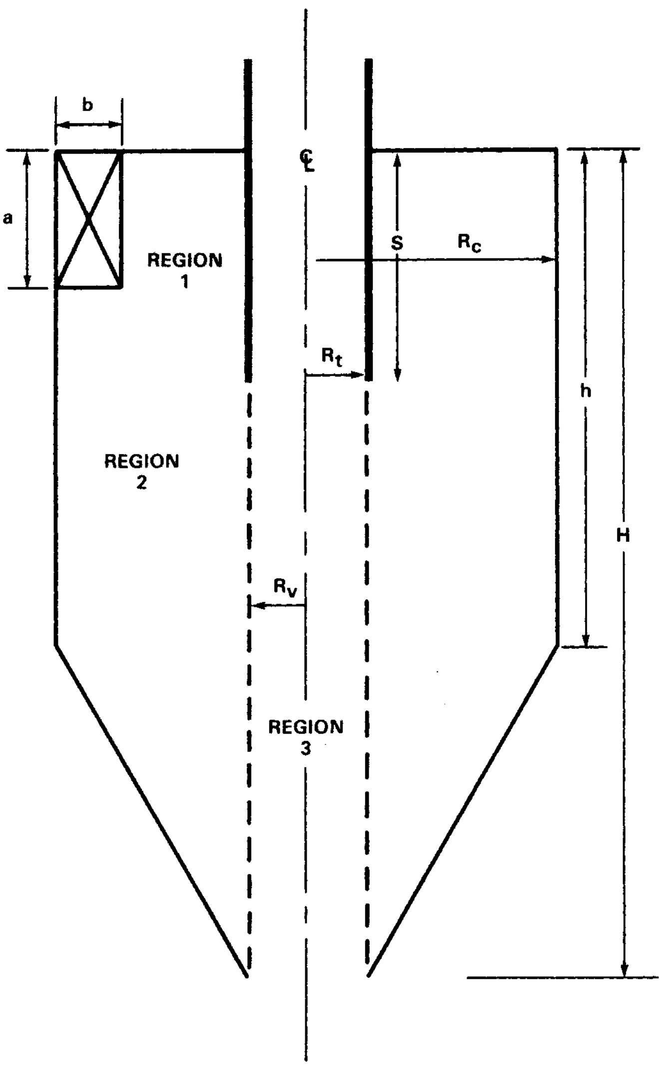

Figure 1. Geometry of a conventional reverse flow cyclone.

INTRODUCTION

The recent emphasis on the development of coal-based technologies for producing electric power has created a renewed interest in cyclone separators. In particular, proposed combined cycle power plants employ cyclones to remove the particulate efflux prior to expansion through the gas turbine. Because of the sensitivity of high speed turbine blades to erosion, the ability to predict and optimize cyclone performance is crucial to this technology. Unfortunately, the necessary design tools are not available.

Historically, cyclones were characterized by a cut size (d_{pc}) which was defined to be the particle size for which the cyclone efficiency was 50 percent. This cut size can be approximately computed by equating the centrifugal force on a particle (due to its angular velocity) to the drag on the particle due to the radial gas velocity. (see for example Stairmand 1951, Stern, et al 1955). Unfortunately, this procedure does not allow prediction of the shape of the grade-efficiency curve. Nonetheless, this simple approach can be used to scale experimental data for geometrically similar cyclones. (Stairmand, 1951; Lapple, 1951) Deviations from this model have been attributed to re-entrainment, particle bounce, and gas by-passing (sneakage). Unfortunately, new cyclone designs must be tested before this technique can be employed.

A recent paper by Leith and Licht (1972) proposed an improved model which recognized the inherently turbulent nature of cyclones. Consequently, their model, which incorporates turbulent back-mixing of the suspended particles, provides reasonable agreement with data. However, two features of their model are inadequate. First, while the distribution of gas residence times within the cyclones is recognized, only the average residence time is used in their analysis. The shorter residence times can result in a significant carry-over of particles and a concomitant loss in cyclone efficiency.

Second, their model is not consistent with the actual gas flow pattern. The assumption that the gas is uniformly mixed across a cross-section of the cyclone and becomes progressively cleaner as the residence time increases, ignores the reverse flow nature of the cyclone. This assumption would be justified in an axial flow device, but if turbulent mixing is an important factor in determining cyclone efficiency, then the interchange of particles between the upflowing and downflowing sections of the cyclone must be included.

In the present paper, a three-region model is proposed for flow within the cyclone. The three regions are the entrance region, the downflow (or annular) region and the upflow (or core) region. Turbulent mixing is assumed to be effective of maintaining uniform radial concentration profiles in each region, and particle interchange is allowed between the annular and core regions. Consequently, the model includes particle by-pass caused by both short gas residence times and mixing into the core region.

CYCLONE MODEL

The gas flow within a conventional cyclone is inherently three-dimensional and turbulent. In the absence of an analytic model for this complex flow (or extensive data), detailed calculations of exact particle trajectories are impossible and approximate techniques must be employed to model the performance of cyclones.

A model is desired which includes the following features:

- Includes cyclone geometry
- Recognizes importance of turbulent mixing
- Provides for a distribution of gas-residence times
- Does not assume the core and annular region are well mixed
- Allows exchange of particles between the core and annular region

The simplest model incorporating these features is a three-region model. The cyclone is conceptually separated into the following regions (Figure 2):

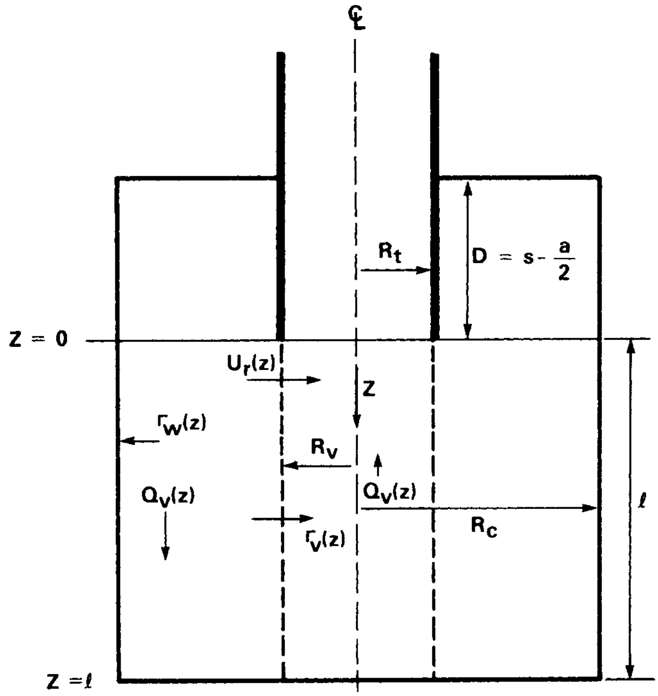

Figure 2. Modified cyclone geometry for analysis.

1) The outer annulus of the cyclone in which the gas velocity is essentially downward (regions 1 and 2)

2) The core of the cyclone in which the gas velocity is essentially upward (region 3)

For analytic convenience, the downflow region is further subdivided into the region above the exhaust tube (1) and the region below it (2).

To further simplify the analysis, the conventional cyclone geometry (see Figure 1) will be modified to a right circular cyclinder (Figure 2). The cyclone radius (R_c) and exit tube radius (R_t) are unchanged. The engagement length of the modified cyclone (D) is equal to the average engagement length ($D = S - a/2$) where S is the actual engagement length of the cyclone and a is the axial extent of the inlet. The length of the model cyclone below the exit tube is equal to the length of the actual cyclone below the exit tube. If this length is longer than the natural turning length of the cyclone as given by Alexander (1949)

$$l = 7.3\ R_t\left(\frac{R_c^2}{ab}\right)^{1/3} \tag{1}$$

then that length is used. (It should be noted that the model is not extremely sensitive to this parameter)

In each region, turbulent mixing is assumed to maintain uniform radial concentration profiles. Thus, conservation of particles in each region requires that

$$\frac{d}{dz}\ [Q_{vo}n_1] = -2\pi R_c\Gamma_w(z) \qquad \text{Region 1} \tag{2}$$

$$\frac{d}{dz}\ [Q_v(z)n_2] = -2\pi R_c\Gamma_w(z) - 2\pi R_v(z)\Gamma_v(z) \qquad \text{Region 2} \tag{3}$$

$$-\frac{d}{dz}\ [Q_v(z)n_3] = 2\pi R_v(z)\Gamma_v(z) \qquad \text{Region 3} \tag{4}$$

where $Q_v(z)$ is the axial volume flow rate in each region (in region 1, the axial volume flow rate is equal to the total volume flow rate of the cyclone, Q_{vo}), $\Gamma_w(z)$ is the particle flux to the cyclone wall, $R_v(z)$ is the radius of the core region and $\Gamma_v(z)$ is the flux of particles from the annular region to the core region (2 → 3).

The radial particle velocity (U_{pw}) at the cyclone wall (where the radial gas velocity is zero) can be directly computed from a balance between inertial and drag forces (assuming Stokes' drag).

$$6\pi\mu R_p U_{pw}(z) = \frac{\frac{4}{3}\ \pi\rho_p R_p^3 U_{tw}^2}{R_c} \tag{5}$$

where μ is the gas viscosity, R_p is the particle radius, $U_{tw}(z)$ is the tangential gas velocity and ρ_p is the mass density of the particle. Thus

$$U_{pw}(z) = \frac{2\rho_p R_p^2 U_{tw}^2}{9\mu R_c} \tag{6}$$

and the particle flux is given by

$$\Gamma_w = n_1 U_{pw} \tag{7}$$

At the boundary between the core and annular regions, the situation is more complex. The "obvious" course would be to balance the drag force (including the radial gas velocity) with the centrifugal force and thus obtain a particle velocity. However, to compute a particle flux, this velocity must be multiplied by a number density (as in Eq. 7). Which number density should be used: n_2 or n_3?

Instead of answering this question, an alternate approach is adopted. The particle flux between the core and annular regions is assumed to be composed of two components. The radial gas velocity carries particles from the annular region to the core and, at the same time, centrifugal force acts to propel particles from the core into the annular region with velocity U_{pv}. Thus, the particle flux is

$$\Gamma_v = n_2 U_r - n_3 U_{pv} \tag{8}$$

Now, the velocity of particles thrown from the core into the annulus can again be computed from a balance between centrifugal forces and drag forces (neglecting the radial velocity which is included in the transport from the annulus to the core). Thus,

$$U_{pv}(z) = \frac{2\rho_p R_p^2 U_{tv}^2}{9\mu R_v} \tag{9}$$

To complete the model, it is only necessary to specify the flow pattern within the cyclone. A simple picture which incorporates many elements of the actual physical system is as follows:

- The radial velocity into the core region is constant

$$U_r(z) = U_{ro} = \frac{Q_{vo}}{2\pi R_v l} \tag{10}$$

- Thus, the axial flow rate is given by

$$Q_v(z) = Q_{vo}(1 - z/l), \tag{11}$$

- The tangential velocity does not vary axially, (Stairmand, 1951)
- The radial dependence of the tangential velocity is given by a modified form of the free vortex in an inviscid fluid (Alexander, 1949; Caplan, 1968)

$$U_t(r) = U_{tw}(R_c/r)^m \tag{12}$$

where m is between 0.5 and 1.0 and,

- The radius of the core region (which separates up flow from down flow) is equal to that of the exit tube (Caplan, 1968; terLinden, 1949; Leith and Licht, 1972)

$$R_v = R_t \tag{13}$$

[The radius of the maximum separative region is somewhat smaller than the exit tube (Stairmand, 1951).]

With the model thus completed, Eqs. 2 to 4 become a set of coupled, non-constant coefficient, ordinary differential equations. After obtaining analytic solutions and matching the inlet condition, the axial concentration profiles in the various regions are

$$n_1(z) = n_o\exp\left[\frac{-2\pi R_c U_{pw}(z + D)}{Q_v}\right] \tag{14}$$

$$n_2 = n_1(z = 0)\left[1 - \frac{z}{l}\right]^\beta \tag{15}$$

and

$$n_3 = n_1(z = 0)\left[\frac{A - \beta}{C}\right]\left[1 - \frac{z}{l}\right]^\beta \tag{16}$$

where

$$\beta \equiv \frac{1}{2}\ [A - 1 - C] + \frac{1}{2}\ [(C - A - 1)^2 + 4AC]^{1/2} \tag{17}$$

$$A \equiv \frac{2\pi R_c l U_{pw}}{Q_v} \tag{18}$$

and

$$C \equiv \frac{2\pi R_t U_{pv} l}{Q_v} \tag{19}$$

The efficiency of the cyclone is given by

$$\eta = 1 - \frac{n_3(z = 0)}{n_o} \tag{20}$$

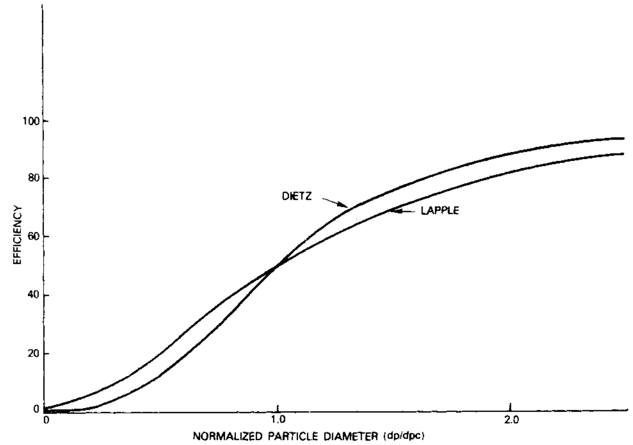

Figure 3. Comparison between present theory and Lapple's model (R_c = 0.1m; assumed m = 0.7).

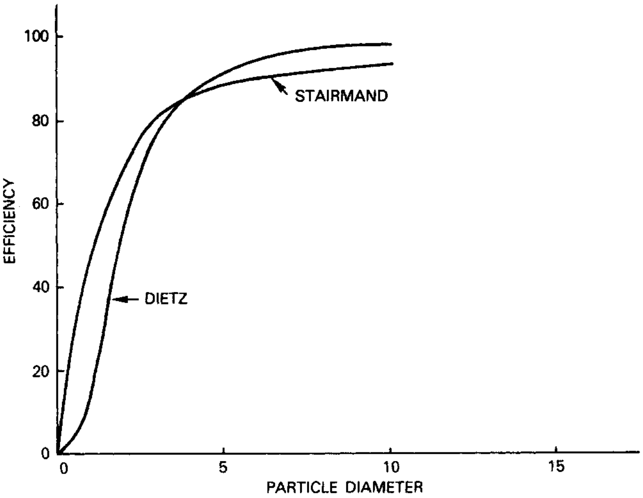

Figure 4. Comparison between present theory and Stairmand's data (assumed m = 0.7).

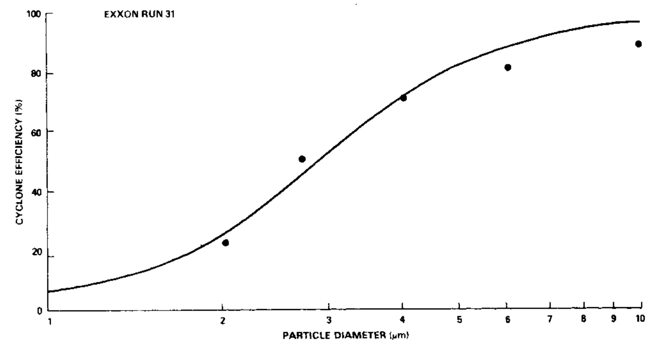

Figure 5. Comparison between theory and experiment for the second stage cyclone at Exxon's miniplant (Run 31) (assumed m = 0.7).

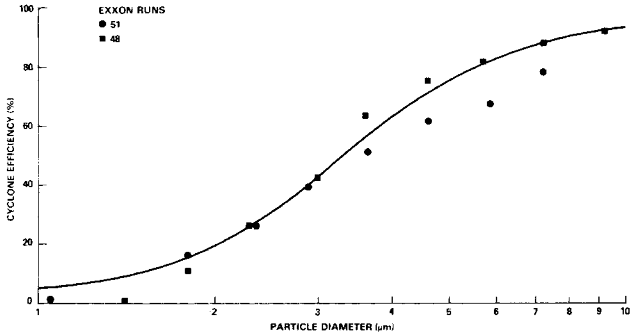

Figure 6. Comparison between theory and experiment for the second stage cyclone at Exxon's miniplant (Runs 48 and 51) (assumed m = 0.7).

TABLE 1. CYCLONE DIMENSIONS*

	Lapple	Stairmand Hi-Efficiency	Exxon Secondary
S	0.127	0.102	0.133m
R_c	0.102	0.102	0.089m
R_t	0.051	0.051	0.044m
h	0.406	0.305	0.356m
H	0.813	0.813	0.711m
a	0.102	0.102	0.102m
b	0.051	0.041	0.044m

* Dimensions defined in Figure 2.

$$= 1 - [K_o - \{K_1^2 + K_2\}^{1/2}] \exp\left[\frac{-2\pi R_c U_{pw} D}{Q_v}\right] \tag{21}$$

where

$$K_o \equiv \frac{R_c U_{pw} + R_v U_{ro} + R_v U_{pv}}{2 R_v U_{pv}} \tag{22}$$

$$K_1 \equiv \frac{R_v U_{pv} - R_v U_{ro} - R_c U_{pw}}{2 R_v U_{pv}}$$

$$= \frac{1}{2}\left[1 - \left(\frac{R_v}{R_c}\right)^{2n}\left\{1 - \frac{9\mu Q_{vo}}{4\pi l \rho_p R_p^2 U_{tw}^2}\right\}\right] \tag{23}$$

and

$$K_2 \equiv \frac{R_c}{R_v}\frac{U_{pw}}{U_{pv}} = \left(\frac{R_v}{R_c}\right)^{2n} \tag{24}$$

COMPARISON WITH EXPERIMENT

Models for the collection efficiency of cyclone separators have been presented by Stairmand (1951) and Lapple (1951). In each case, cyclone efficiency curves are presented for generic cyclone designs. The cyclone dimensions are listed in Table 1 and the operating conditions are outlined in Table 2. In both cases, the cyclone diameter is 0.2 m (8 in.).

In Figure 3, a comparison is made between Lapple's model and Eq. 21. To make the comparison, the results have been normalized to the cut size (this is the form chosen by Lapple).

TABLE 2. CYCLONE OPERATING CONDITIONS

	Lapple	Stairmand High Efficiency	Exxon Run 31	Exxon Run 48	Exxon Run 51
Pressure (Pascal)	1.0×10^5	1.0×10^5	6.0×10^5	9.3×10^5	1.0×10^5
Temperature (°C)	20°	20°	948	875	871
Air Density (kg/m^3)	1.0	1.0	1.7	2.8	2.8
Volume Flow Rate (m^3/s)	0.079	0.064	0.247	0.110	0.110
Cyclone Inlet Velocity (m/s)	15.2	15.2	54.8	24.4	24.4
Particle Mass Density (kg/m^3)	2.0×10^3	2.0×10^3	2.5×10^3	2.5×10^3	2.5×10^3
Gas Viscosity (Pa-s)	2.0×10^{-5}	2.0×10^{-5}	4.8×10^{-5}	4.6×10^{-5}	4.6×10^{-5}

The agreement in the curves is good. In addition, if Lapple's formula for the cut size is employed, the predicted cut-size of 3.09 μm compares well with the model's prediction of 3.01 μm.

In Figure 4, the model is compared with Stairmand's data for a high-efficiency cyclone. The agreement, though not as good, is satisfactory.

A recent General Electric report (FE-2357-70, 1980) to the Department of Energy documents fractional-efficiency data for high-temperature, high-pressure cyclones collecting the particulate efflux from coal-fired pressurized fluidized bed combustors. Data is reported for cyclones on the Exxon miniplant facility in Linden, New Jersey and the National Coal Board/Coal Utilization Research Laboratory facility in Leatherhead, England. Both sets of data are compared with the theory presented in this paper. Figures 5 and 6 show representative fractional efficiency data for the second-stage cyclone at the Exxon miniplant facility. Again, the cyclone dimensions are listed in Table 1 and the operating conditions are outlined in Table 2.

The theoretical efficiencies for these cyclones are presented on the same graphs. As Runs 48 and 51 were performed under identical experimental conditions, an estimate of the experimental scatter can be obtained from Figure 6 in which the data from both runs are presented. In view of this scatter, the agreement between theory and experiment is excellent.

SUMMARY

In the present paper a new, analytic model is developed for the collection efficiencies of reverse flow cyclones. Within each of three regions, turbulent mixing is assumed to maintain uniform radial concentration distributions. In addition, by incorporating a distribution of gas residence times, the loss in efficiency due to gas bypassing is also modeled. Although this new model does not include particle bounce or re-entrainment, the resultant analytic equation provides good agreement with data for high efficiency cyclones and, consequently, may provide a needed design tool for characterizing new cyclone configurations.

ACKNOWLEDGMENT

This work was performed under DOE Contract No. EX-76-C-01-2357 to the General Electric Energy Systems Program Department by Corporate Research and Development.

NOTATION

a = axial extent of inlet
A, C = intermediate variables for computing β
b = radial extent of inlet
D = effective engagement length
K_o, K_1, K_2 = intermediate variables in computing efficiency
l = effective cyclone length
m = vortex strength ($0.5 < m < 1.0$)
n = number density
Q_v = volume flow rate
R_c = radius of cyclone
R_p = particle radius
R_t = radius of exit tube
R_v = radius of vortex
S = engagement length of cyclone
U_t = tangential gas velocity (U_{tw}, wall; U_{tv}, vortex)
U_p = particle velocity (U_{pw}, wall; U_{pv}, vortex)

Greek Letters

β = exponent in solution
η = efficiency
ρ_p = particle mass density
μ = gas viscosity
Γ = particle flux

LITERATURE CITED

Alexander, R. McK., "Fundamentals of Cyclone Design and Operation," Proc. Aust. Inst. Mining and Met., 152-153, p. 203 (1949).

Caplan, K. J., "Source Control by Centrifugal Force and Gravity," *Air Pollution*, **III**, p. 359, A. C. Stern, ed., Academic Press, New York (1968).

Lapple, C. E., "Process Uses Many Collector Types," *Chem. Eng.*, **58**, 144 (1951).

Leith, D., and W. Licht, "The Collection Efficiency of Cyclone Type Particle Collectors—A New Theoretical Approach," *AIChE Symp. Ser.*, No. 126, **68**, p. 196 (1972).

"Advanced Cleanup Device Performance Design Report Volume A: Cyclone Theory and Data Correlation," General Electric, FE-2357-70, PFB Coal Fired Combined Cycle Development Program, Prepared for US Department of Energy under Contract No. DE-AC21-76ET10377 (1980).

Stairmand, C. J., "The Design and Performance of Cyclone Separators," *Trans. Inst. Chem. Eng.*, **29**, p. 356 (1951).

Stern, A. C., K. J. Caplan, and P. D. Bush, "Cyclone Dust Collectors," Am. Petrol Inst., New York (1955).

terLinden, A. J., "Investigations into Cyclone Dust Collectors," IME Proc. 1960, 233 (1949).

Manuscript received October 3, 1979; revision received October 14, 1980 and accepted January 20, 1981.